

ÉRETTSÉGI VIZSGA • 2018. május 8.

**MATEMATIKA
ANGOL NYELVEN**

**EMELT SZINTŰ
ÍRÁSBELI VIZSGA**

**JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ**

EMBERI ERŐFORRÁSOK MINISZTERIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
 - correct calculation: *checkmark*
 - principal error: *double underline*
 - calculation error or other, not principal, error: *single underline*
 - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
 - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
 - unintelligible part: *question mark and/or wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
 2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
 3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
 4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
 5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
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6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
 7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
 8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
 9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
 10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**:
addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$,
replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e , finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
 11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
 12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
 13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
 14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)		
The class midpoints (in kilogram) are: 54.5; 58.5; 62.5; 66.5; 70.5; 74.5; 78.5.	1 point	<i>This point is also due if these numbers are used in the calculations.</i>
Use the above to calculate the mean: $\frac{2 \cdot 54.5 + 3 \cdot 58.5 + 4 \cdot 62.5 + 11 \cdot 66.5 + 9 \cdot 70.5 + 6 \cdot 74.5 + 5 \cdot 78.5}{40} =$	1 point	<i>These 2 points are also due if the candidate does not detail their calculations but obtains the correct answer by calculator.</i>
= 68.5 (kg).	1 point	
The standard deviation: $\sqrt{\frac{2 \cdot 14^2 + 3 \cdot 10^2 + 4 \cdot 6^2 + 11 \cdot 2^2 + 9 \cdot 2^2 + 6 \cdot 6^2 + 5 \cdot 10^2}{40}} \approx$	1 point	<i>These 2 points are also due if the candidate does not detail their calculations but obtains the correct answer by calculator.</i>
≈ 6.39 (kg).	1 point	
Total:	5 points	

1. b)		
Among the 40 students, there are 9 “lightweights” and 5 “heavyweights”.	1 point	<i>These 2 points are also due if the correct reasoning is reflected only by the solution.</i>
There are $\binom{9}{3}$ different ways to select the three lightweights, and $\binom{5}{2}$ different ways to select the two heavyweights.	1 point	
The total number of different possibilities is $\binom{9}{3} \cdot \binom{5}{2} = 840$.	1 point	
Total:	3 points	

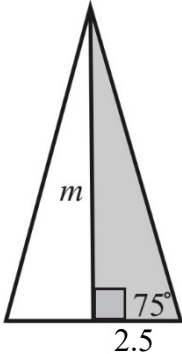
1. c)		
The sum of the five grades is $(3.2 \cdot 5 =) 16$.	1 point	
(As there may only be two grades, at most, smaller than the median) there are 2 twos	1 point	
and so the five grades are 2, 2, 3, 4, 5 (and this is the only possible set of grades).	1 point	
The average absolute deviation from the mean is $\frac{ 2-3.2 + 2-3.2 + 3-3.2 + 4-3.2 + 5-3.2 }{5} =$	1 point	
$= 1.04$.	1 point	
Total:	5 points	

2. a)		
Let α be the smallest angle of the quadrilateral. The four angles then are α , 3α , 9α és 27α .	1 point	
$40\alpha = 360$	1 point	
$\alpha = 9$	1 point	
The angles of the quadrilateral are 9° , 27° , 81° , and 243° (such quadrilateral does, in fact, exist).	1 point	
Total:	4 points	

2. b)		
(Assume the polygon has n sides.) The sum of the interior angles is $(n-2) \cdot 180^\circ$,	1 point	<i>These points are also due if the correct reasoning is reflected only by the solution.</i>
also $\frac{[2 \cdot 143^\circ + (n-1) \cdot 2^\circ] \cdot n}{2}$.	1 point	
$\frac{[2 \cdot 143 + (n-1) \cdot 2] \cdot n}{2} = (n-2) \cdot 180$.	1 point	
$143n + n^2 - n = 180n - 360$	1 point	
$n^2 - 38n + 360 = 0$	1 point	
The two solutions of the equation are 18 and 20.	1 point	
Check: 20 is not a solution, as the polygon in that case would not be convex, its greatest angle being 181° .	1 point	
An 18-gon is a correct solution, the polygon would be convex, as the greatest angle is 177° . (Such polygon does exist, $143 + 145 + 147 + \dots + 177 = 2880 = 16 \cdot 180$).	1 point	
Total:	8 points	

3. a)		
Rearrange the inequality: $x^2 - 5x - 50 < 0$.	1 point	
The roots of the equation $x^2 - 5x - 50 = 0$ are 10 and -5 .	1 point	
As the coefficient of the quadratic term is positive,	1 point	<i>This point is also due for the correct diagram.</i>
the solution of the inequality is $-5 < x < 10$.	1 point	
Total:	4 points	

3. b)		
The domain of the inequality is $x > 0$.	1 point	
(Using the rules of logarithms: $2\log_3 x - \log_9 81x \leq 1$	1 point	
$2\log_3 x - (\log_9 81 + \log_9 x) \leq 1$	1 point	
$2\log_3 x - \log_9 x \leq 3$	1 point	
$2\log_3 x - \frac{\log_3 x}{\log_3 9} \leq 3$	1 point	
$4\log_3 x - \log_3 x \leq 6$	1 point	
$\log_3 x \leq 2$ ($= \log_3 9$)	1 point	
As the base-3 logarithm function is strictly monotone increasing, here $x \leq 9$.	1 point	
Comparing this with the domain, the solution is: $0 < x \leq 9$.	1 point	
Total:	9 points	

4. a)		
<p>(The area of the regular 12-gon may be obtained by dividing it into 12 congruent isosceles triangles.) The base of such a triangle is 5 m, the base angles are 75°,</p>		1 point
the height that belongs to the base is $2.5 \cdot \tan 75^\circ$ (≈ 9.33) (m).		1 point
The area of one such triangle is $\frac{5 \cdot 2.5 \cdot \tan 75^\circ}{2}$ (≈ 23.3) (m^2), the area of the 12-gon is $(12 \cdot 2.5^2 \cdot \tan 75^\circ \approx)$ 280 m^2 .		1 point
The volume of the straight prism part is $8 \cdot 12 \cdot 2.5^2 \cdot \tan 75^\circ$ (≈ 2240) (m^3).		1 point
The volume of the top pyramid is $\frac{3 \cdot 12 \cdot 2.5^2 \cdot \tan 75^\circ}{3}$ (≈ 280) (m^3).		1 point
The volume of the tent is approximately $(2240 + 280 =)$ 2520 m^3 .		1 point
As $\frac{2520}{200} = 12.6$,		1 point
a minimum of 13 heaters are required in winter.		1 point
Total:		8 points

These 3 points are also due if the candidate correctly calculates the area of the dodecagon with the formula found in the four-digit data tables.

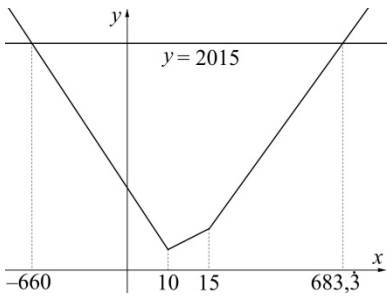
A different answer may be accepted as long as it is rounded reasonably and correctly.

4. b)		
No more than one mistake means either 1 or no mistake at all.	1 point	<i>These 2 points are also due if the correct reasoning is reflected only by the solution.</i>
The probability of successfully catching a club is 0.997.	1 point	
The probability of missing no more than once: $0.997^{72} + \binom{72}{1} \cdot 0.997^{71} \cdot 0.003 \approx$	2 points	
$(\approx 0.805 + 0.175) \approx 0.98$.	1 point	<i>Do not award this point if the solution is not rounded or rounded incorrectly.</i>
Total:		5 points

II.

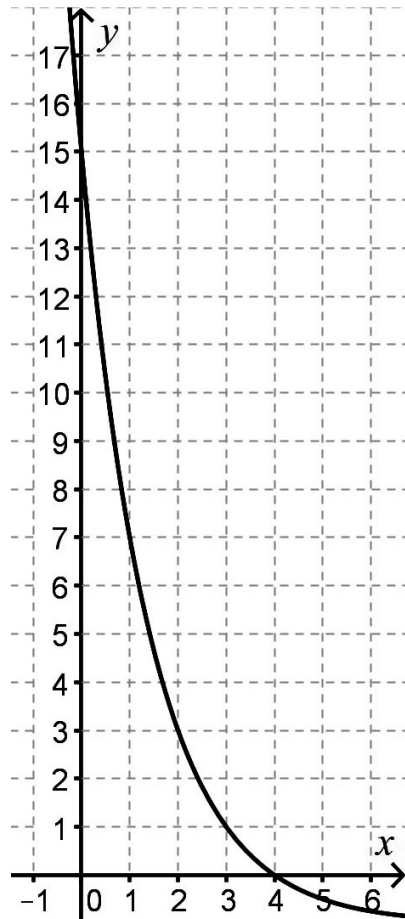
5. a)		
The solution of the inequality in the interval $[0; 2\pi]$: $\frac{\pi}{3} < x < \frac{5\pi}{3}$.	2 points	<i>Award maximum score if the candidate obtains the correct solution by substituting every one of the seven integers in the interval $[0; 2\pi]$.</i>
(As $\frac{\pi}{3} \approx 1.05$ and $\frac{5\pi}{3} \approx 5.24$) the integer solutions of the inequality are 2, 3, 4, and 5.	1 point	
Total:	3 points	

5. b) Solution 1		
The expressions within the absolute values change their signs at 10 and 15 respectively, thus resulting in three different cases.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
If $x < 10$, the inequality turns into: $20 - 2x + 15 - x < 2015$, that is $-660 < x$.	1 point	
Comparing this to the condition $x < 10$ one obtains 669 suitable integers (659 negative integers, zero, and 9 positive integers).	1 point	<i>Comparing this to the condition $x < 10$, the solution in the set of real numbers is the interval $]-660; 10[$.</i>
If $10 \leq x < 15$, then $2x - 20 + 15 - x < 2015$, that is $x < 2020$.	1 point	
Hence all 5 integers meeting the initial condition (10, 11, 12, 13, 14) are solutions.	1 point	<i>Comparing this to the condition $10 \leq x < 15$ the solution in the set of real numbers is the interval $[10; 15[$.</i>
If $15 \leq x$, then $2x - 20 + x - 15 < 2015$, that is $x < \frac{2050}{3} = 683\frac{1}{3}$.	1 point	
There are 669 integer meeting the initial condition ($683 - 15 + 1 = 669$).	1 point	<i>Comparing this to the condition $15 \leq x$, the solution in the set of real numbers is the interval $\left[15; 683\frac{1}{3}\right]$.</i>
The total number of integer solutions of the inequality (the sum of the above) $2 \cdot 669 + 5 = 1343$.	1 point	<i>The solution of the inequality in the set of real numbers is the interval $\left]-660; 683\frac{1}{3}\right[$. This interval contains 1343 integers, so the original inequality also has that many integer solutions.</i>
Total:	8 points	

5. b) Solution 2		
Consider the function $f: \mathbf{R} \rightarrow \mathbf{R}, f(x) = 2x - 20 + x - 15 .$	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$f(x) = \begin{cases} -3x + 35, & \text{if } x < 10 \\ x - 5, & \text{if } 10 \leq x < 15 \\ 3x - 35, & \text{if } x \geq 15 \end{cases}$	3 points	
The line $y = 2015$ intersects the graph of f in two points: $(-660; 215)$ and $(683\frac{1}{3}; 2015)$.	1 point	
	1 point	
(Between the two points of intersection points of the graph of f are always below the line $y = 2015$, while all other, not common, points are above it, so) the original inequality has as many integer solutions as many integers there are in the interval $\left] -660; 683\frac{1}{3} \right[$	1 point	
The number of integer solutions is therefore 1343.	1 point	
Total:	8 points	

5. c)

The graph of the function $x \mapsto \left(\frac{1}{2}\right)^{x-4} - 1$ is:



1 point

This point is also due if the correct answer is given without a diagram.

(Compute the values of the function for each non-negative integer within the domain up to the zero of the strictly monotone decreasing function:
 $f(0) = 15, f(1) = 7, f(2) = 3, f(3) = 1, f(4) = 0$.)

2 points

Award 1 point if there is one error, 0 points if there are multiple errors.

There are 16 suitable gridpoints on the line $x = 0$, another 8 gridpoints on line $x = 1$, etc.

1 point

This point is also due if the correct reasoning is reflected only by the solution.

There are a total $(16 + 8 + 4 + 2 + 1 =)$ 31 gridpoints within the given section of the coordinate system.

1 point

Total:

5 points

6. a)		
(1) true (2) false (3) false (4) false (5) true	3 points	<i>Award 2 points for 4 correct answers, 1 point for 3 correct answers, 0 points for less than 3 correct answers.</i>
Total:	3 points	

6. b)		
(There are two possible answers for each of the five independent questions, so) the total number of different possible ways to answer the questions is $2^5 = 32$.	2 points	
As there are 34 students in the class, (according to the Pigeonhole Principle) there must be two among them who gave exactly the same answers for each question.	2 points	
Total:	4 points	

6. c)		
By giving the correct answer to each question, one may receive $1 + 2 + 3 + 4 + 5 = 15$ points.	1 point	
Giving the wrong answer for question n this total score is reduced by $2n$, which is an even number.	2 points	
So, the possible scores must always be odd.	1 point	
Total:	4 points	

Note: Award maximum score if the candidate gives the correct answer by examining all possible cases.

6. d)		
Apart from the order of the terms, there are three ways to write 39 as a sum of odd integers.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$13 + 13 + 13$, these terms can only be written in one order.	1 point	
$15 + 13 + 11$, these terms can be written in six different orders.	1 point	
$15 + 15 + 9$, these terms can be written in three different orders.	1 point	
There are 10 different possible ways to make 39 points, meeting all of the given conditions.	1 point	
Total:	5 points	

7. a)		
$f'(x) = 2ax + b$, and so	1 point	
$\left. \begin{array}{l} 4a + b = 6 \\ 12a + b = 2 \end{array} \right\}$	2 points	
The solution of the equation system: $a = -0.5$ and $b = 8$.	1 point	
$\int_0^2 (-0.5x^2 + 8x + c) dx = \left[-\frac{0.5}{3}x^3 + 4x^2 + cx \right]_0^2 =$	1 point	
$= \frac{44}{3} + 2c.$	1 point	
The equation $\frac{44}{3} + 2c = \frac{50}{3}$ yields $c = 1$.	1 point	
Total:	7 points	

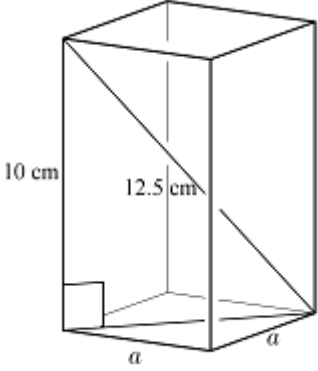
7. b) Solution 1		
Of the lines passing through the point $P(0; 35)$, the one with equation $x = 0$ is not a tangent to the parabola (as it is parallel to the axis of the parabola),	1 point	
and so we will find the equation of the tangent in the form of $y = mx + 35$ (here m is the gradient of the line).	1 point	
The line is tangent to the parabola if and only if the equation system $\left. \begin{array}{l} y = mx + 35 \\ y = -\frac{1}{2}x^2 + 8x + 3 \end{array} \right\}$ has a single solution of an ordered pair of real numbers.	1 point	
Express y from the first equation, substitute it into the second, and rearrange: $\frac{1}{2}x^2 + (m - 8)x + 32 = 0.$	1 point	
The equation system has a single solution if this quadratic equation has a single solution, i.e. its discriminant is zero.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$(m - 8)^2 - 64 = 0.$	1 point	
This yields $m = 0$, and $m = 16$.	1 point	
There are two possible tangents: $y = 35$, and $y = 16x + 35$.	2 points	
Total:	9 points	

7. b) Solution 2		
Of the lines passing through the point $P(0; 35)$, the one with equation $x = 0$ is not a tangent to the parabola (as it is parallel to the axis of the parabola),	1 point	

The gradient of the tangent that passes through the point $P(0; 35)$ and touches the parabola in the point $Q(q; -0,5q^2 + 8q + 3)$ is $\frac{\left(-\frac{1}{2}q^2 + 8q + 3\right) - 35}{q}$ (where $q \neq 0$).	1 point	
The gradient of the tangent drawn to the parabola in point Q is also equal to the derivative of the function $f(x) = -\frac{1}{2}x^2 + 8x + 3$ ($x \in \mathbf{R}$) at q .	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
One has to solve the equation $\frac{\left(-\frac{1}{2}q^2 + 8q + 3\right) - 35}{q} = -q + 8$.	1 point	
Rearranging the equation: $q^2 = 64$.	2 points	
$q = 8$ or $q = -8$ (so $-q + 8 = 0$ or $-q + 8 = 16$).	1 point	
There are two possible tangents: $y = 35$, and $y = 16x + 35$.	2 points	
Total:	9 points	

7. b) Solution 3

The equation of the line that passes through the point $P(0; 35)$ and has normal vector $(A; B)$ is $Ax + By = 35B$.	1 point	
The common points of the line and the parabola are determined by the solutions of the equation system: $\left. \begin{array}{l} y = -\frac{1}{2}x^2 + 8x + 3 \\ Ax + By = 35B \end{array} \right\}$	1 point	
Express y from the first equation, substitute it into the second, and rearrange: $\frac{1}{2}Bx^2 - (A + 8B)x + 32B = 0$.	1 point	
If $B = 0$, (the equation is linear and) the line passing through point P is parallel to the axis of the parabola, hence not a tangent.	1 point	
If $B \neq 0$, the equation is quadratic. The line through point P is tangent to the parabola if and only if the quadratic equation has one real root, i.e. the discriminant is zero:	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$(A + 8B)^2 - 64B^2 = 0$.	1 point	
This yields $A = 0$, and $A = -16B$.	1 point	
There are two tangents. Their equations are $y = 35$ (in case $B = 1$), and $-16x + y = 35$.	2 points	
Total:	9 points	

8. a)		
<p>Correct diagram, complete with data:</p> 	1 point	<i>This point is also due if the correct answer is given without a diagram.</i>
<p>Use the Pythagorean Theorem to find the length of the diagonal of the base: $\sqrt{12.5^2 - 10^2} = 7.5$ (cm).</p>	1 point	
<p>The length of the base edge: $a = \frac{7.5}{\sqrt{2}}$ (≈ 5.3) (cm).</p>	1 point	
<p>The sum of the areas of the two square faces ($2a^2 =$) 56.25 cm².</p>	1 point	
<p>The lateral surface area ($4 \cdot a \cdot 10 =$) $4 \cdot \frac{7.5}{\sqrt{2}} \cdot 10$ ($=150\sqrt{2}$) cm² (≈ 212.13 cm²).</p>	1 point	
<p>The surface area of the cuboid is ($56.25 + 212.13 \approx$) 268 cm².</p>	1 point	
Total:	6 points	

8. b)		
(Let the length of the inside edges of the square faces be x dm, the length of the other edges be y dm.) $x^2y = 288$ (dm ³),	1 point	
the inside surface area of the tank is $2x^2 + 3xy$ (dm ²).	2 points	<i>1 point for the total area of the square faces, 1 point for the total area of the other three faces.</i>
Express y from the first equation, and substitute it into the second. The interior area is $2x^2 + \frac{864}{x}$ (dm ²).	1 point*	
Find the minimum of the function $f: \mathbf{R}^+ \rightarrow \mathbf{R}, f(x) = 2x^2 + \frac{864}{x}$.	1 point*	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The derivative of f (which is differentiable over its domain) is $f'(x) = 4x - \frac{864}{x^2}$.	1 point*	
f may have a minimum wherever its derivative is 0, that is $4x - \frac{864}{x^2} = 0$.	1 point*	
From which $x = 6$.	1 point	
The second derivative at $x = 6$ is positive, so here f has a (local and also global) minimum.	1 point	<i>This point is also due if the candidate refers to the change of sign in the first derivative.</i>
The (inner) length of the edges of the square faces of the tank is 6 dm, the other edges are $\left(\frac{288}{6^2} = \right) 8$ dm.	1 point	
Total:	10 points	

The 4 points marked by * may also be given for the following reasoning:

Refer to the inequality between the arithmetic and geometric means.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$2x^2 + \frac{864}{x} = 2x^2 + \frac{432}{x} + \frac{432}{x} \geq 3 \cdot \sqrt[3]{2x^2 \cdot \frac{432}{x} \cdot \frac{432}{x}} = 72$.	2 points	
Equation only if $2x^2 = \frac{432}{x}$.	1 point	

9. a)		
One ticket will certainly not be enough. (If the four numbers not marked on his ticket are all winners, he will only get one of the winning numbers right.)	1 point	
If he plays with two tickets marking, for example, the numbers (1; 2; 3; 4; 5) on one of them and (5; 6; 7; 8; 9) on the other, then he will get at least three of the winning numbers on at least one of these tickets.	2 points	
Therefore, the minimum number of tickets is two.	1 point	
Total:	4 points	

9. b) Solution 1		
Assume that Zoli has already marked his five numbers.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
There are a total $\binom{9}{5}$ (= 126) possible ways for Dóra to fill her ticket.	1 point	
They will have exactly four numbers in common, if four out of Dóra's five numbers come from among those also selected by Zoli, while the other one comes from among those not selected by Zoli.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The number of favourable cases is therefore $\binom{5}{4} \cdot \binom{4}{1}$ (= 20).	1 point	
The probability is $p = \frac{20}{126} = \frac{10}{63}$ (≈ 0.159).	1 point	
Total:	5 points	

9. b) Solution 2		
There are $\binom{9}{5} \cdot \binom{9}{5}$ (= 15 876) different ways to fill in two tickets.	1 point	
There are $\binom{9}{4}$ (= 126) different ways to select the four numbers they both mark.	1 point	
Dóra may select her fifth number out of the remaining 5, while Zoli has only 4 choices left (as this is the one number that is different).	1 point	
The number of favourable cases: $\binom{9}{4} \cdot 5 \cdot 4$ (= 2520).	1 point	
The probability is $p = \frac{2520}{15\,876} = \frac{10}{63}$ (≈ 0.159).	1 point	
Total:	5 points	

9. c)		
$3780 = 2^2 \cdot 3^3 \cdot 5 \cdot 7$,	1 point	
so 5 and 7 must be among the numbers marked.	1 point	
The number 9 must also be marked, otherwise the exponent of 3 could be no more than two (in the product of 3 and 6)	1 point	
The product of the remaining two numbers must be a multiple of $2^2 \cdot 3 (= 12)$.	1 point	
If the fourth number marked is 3, the last number can be either 4 or 8.	1 point	<i>Out of the numbers 1, 2, 3, 4, 6, 8 two must be selected, such that their product is a multiple of 12: 2-6, 3-4, 3-8, 4-6, 6-8.</i>
If the fourth number marked is 6, the last number can be either 2 or 4 or 8.	1 point	
The number of such tickets is therefore five.	1 point	
Total:	7 points	

Note: If the candidate lists the possible combinations proving that they are all correct (while not proving that there aren't any other correct solutions), 1 point should be awarded for each correct answer. If the candidate also lists incorrect answers, deduct 1 point per incorrect answer. (Be aware, though, that the total score given must not be negative!)